Monetary Policy Conduct: A Hybrid Framework

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November 2008

Technical Appendix

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Technical Appendix

Appendix A: The Model Setting

We construct a model that is a variant of a dynamic New-Keynesian model applied to a small-open economy, following Clarida et al (2002) and Gali and Monacelli (2005). In order to make this paper self-contained, key structural equations are presented in this Technical Appendix.

The model has three sectors: 1) a continuum of profit-maximizing, monopolistically-competitive firms (owned by consumers who include their shares in their portfolios) operating a constant return-to-scale technology and making staggered price decisions in the spirit of Calvo (1983); 2) an infinitely-lived representative household which maximizes a utility function defined over a composite consumption-good and labour supply; and 3) a central bank which sets the monetary policy through an interest rule that targets both the price level and the inflation rate in a hybrid formula.

Firms' Problem

Production technology

This model has a continuum of identical monopolistically-competitive firms. As in, for example, Gali and Monacelli (2005), the production function for firm i that produces a differentiated good Y_i is

$$Y_t(i) = A_t N_t(i), \tag{1}$$

where $i \in [0, 1]$, $Y_t(i)$ and $N_t(i)$ are the firm i's specific output and labour input, respectively. $a_t = \log(A_t)$ is a total-factor productivity index driven by an $AR(1)$ exogenous stochastic process, $\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{a,t}$,where $\varepsilon_{a,t}$ is a white noise with mean 0 and variance σ_ϵ^2.

The cost minimization problem leads to the expression of real marginal cost in terms of home prices as $MC_t = \frac{W_t(1-\tau)}{A_t P_{H,t}}$. Hence, the log of real marginal cost, which is common across domestic firms, is given by

$$\widehat{mc}_t = \hat{w}_t - \nu - \hat{p}_{H,t} - \hat{a}_t, \tag{2}$$

where $\hat{p}_{H,t}$ and $\hat{w}_t$ stand respectively for the deviations of domestic price and of wage rate from their steady-state values. $\nu = -\log(1-\tau)$, where τ is an employment subsidy created to exactly compensate for the monopolistic competition distortion. The employment subsidy exactly offsets the combined effects of the firm's market power and the terms-of-trade distortions in the steady-state. In this case, there is only one effective distortion left in the small-open-economy model, namely sticky prices.

Let Y_t define the aggregate index for domestic output and N_t, the aggregate employment. Y_t and N_t can be expressed in terms of an individual firm's output as follows

$$Y_t = [\int_0^1 Y_t(i)^{\frac{\xi-1}{\xi}} di]^{\frac{\xi}{\xi-1}} \text{ and } N_t = \int_0^1 N_t(i)\, di = \int_0^1 \frac{Y_t(i)}{A_t}\, di$$

where $\xi > 1$ is the elasticity of substitution among goods within each category. Moreover, defining $Z_t = \int_0^1 \frac{Y_t(i)}{Y_t} di$ yields $N_t = \frac{Y_t Z_t}{A_t}$. In loglinear form (up to a first order approximation), aggregate output reduces to

$$\hat{y}_t = \hat{a}_t + \hat{n}_t, \tag{3}$$

where the variables $\hat{y}_t, \hat{a}_t$ and $\hat{n}_t$ represent the deviations of output, total-factor productivity and employment from a symmetric steady-state.

Price Setting, Price Indexation and the Introduction of the Imperfect Pass-Through

Price-setting behaviour follows Calvo (1983) and Yun (1996) in that only a fraction $(1-\psi)$ of firms adjust their prices each period. Indeed, firms are not allowed to change their prices unless they receive a signal allowing them to re-optimize prices. Following Christiano, Eichenbaum and Evans (2005), prices set by firms that do not receive a random price-change signal are indexed to past inflation.[1] Furthermore, Christiano et al. (2005) assume that prices are fully indexed to past inflation, but empirical models that allow for partial indexation (following Smets and Wouters, 2003) often find that the best-fitting value for the degree price indexation is positive but less than one. The partial indexation allows us to have some inflation inertia, leeway which can make the model more robust for policy and welfare analysis, especially if we are interested in welfare evaluation of inflation costs. Erceg, Henderson and Levine (2000) use indexation to the steady-state inflation rate, allowing them to compute a linearized equation for inflation combining expected future inflation and lagged inflation. This equation differs from the forward-looking inflation process obtained under the standard Calvo model.

Let $P^n_{H,t}$ be the price set by firm i adjusting its price in period t and facing a probability ψ^k of keeping its price unchanged for k periods (for $k = 0, 1, 2, ...$). $P^b_{H,t}$ defines the price chosen by the remaining fraction ψ of firms not optimally adjusting their prices at time t. The (log) price $\hat{p}^b_{H,t}$ is set according to the simple, backward-looking rule $\hat{p}^b_{H,t} = \hat{p}_{H,t-1} + \gamma_p \hat{\Pi}_{H,t-1}$, while the new price must satisfy the following equation

$$P^n_{H,t} = \mu + (1-\beta\psi)\sum_{k=0}^{\infty}(\beta\psi)^k E_t\{mc_{t+k} + P_{H,t+k}\}, \tag{4}$$

where γ_p is the coefficient of price indexation and μ is the steady-state markup.[2] The dynamics of the domestic price index are then given by

$$P_{H,t} = [\psi(P^b_{H,t-1})^{1-\xi} + (1-\psi)(P^n_{H,t})^{1-\xi})]^{\frac{1}{1-\xi}} \tag{5}$$

[1] Price indexation makes the price dispersion between individual prices of the monopolistic firms much smaller compared with constant price-setting behaviour, a factor which has important consequences for monetary-policy evaluation. See Rabanal and Rubio-Ramírez, 2005, for a general discussion about price indexation.

[2] The forward-looking pricing decision is related to the fact that firms that adjust their price in any period do so for a random number of periods. The price is then set as a markup over the average of expected future marginal costs.

which can be loglinearized to obtain an expression for the domestic inflation as follows:

$$\hat{\pi}_{H,t} = \psi\gamma_p\hat{\pi}_{H,t-1} + (1-\psi)(\hat{p}^n_{H,t} - \hat{p}_{H,t-1}). \tag{6}$$

Combining (6) with the differentiated version of (5) yields the aggregate supply equation

$$\hat{\pi}_{H,t} = \frac{\beta}{1+\beta\psi\gamma_p}E_t\{\hat{\pi}_{H,t+1}\} + \frac{\gamma_p}{1+\beta\psi\gamma_p}\hat{\pi}_{H,t-1} + \kappa\overset{\curlywedge}{mc}_t \tag{7}$$

where $\kappa = (1-\beta\psi)(1-\psi)/\psi(1+\beta\psi\gamma_p)$. $\overset{\curlywedge}{mc}_t$ represents the log-deviation of the real marginal cost. Equation (7) shows that the domestic inflation dynamic has both forward-looking and backward-looking components. The real marginal costs faced by the firms are also an important determinant of domestic inflation. Note that with $\gamma_p = 0$, this equation reverts to the standard open-economy supply equation.

Moreover, assuming that the degree of price stickiness ψ is identical across economies, the firms in the rest of the world (ROW) face simple Calvo-style price-setting behaviour. For simplicity and without loss of generality, we assume, throughout our analysis, that the degree of price indexation in the ROW γ^*_p is equal to zero.[3]

Households

The small-open economy is inhabited by a continuum of infinitely-lived households where the representative household seeks to maximize the expected utility

$$E_t\sum_{t=0}^{\infty}\beta^t U(C_t, N_t), \tag{8}$$

where N_t is hours worked and C_t is a composite consumption index defined by

$$C_t = [(1-\alpha)^{\frac{1}{\theta}}(C_{H,t})^{\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}}\,(C_{F,t})^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}. \tag{9}$$

[3]Setting γ^*_p so that it is equal to the domestic price-indexation coefficient (or $\gamma^*_p \neq 0$) does not significantly change the policy-evaluation results.

The elasticity of substitution between the indices of home and foreign goods is given by $\theta > 0$. $C_{H,t}$ is the consumption index of j domestic goods defined by the CES aggregator $C_{H,t} = [\int_0^1 (C_{H,t}(j))^{\frac{\xi-1}{\xi}} dj]^{\frac{\xi}{\xi-1}}$. Likewise, the consumption index $C_{F,t}$ is the index of j imported goods given by $C_{F,t} = [\int_0^1 (C_{F,t}(j))^{\frac{\xi-1}{\xi}} dj]^{\frac{\xi}{\xi-1}}$, where the elasticity of substitution among goods within the two indices ξ is greater than one.

Maximization of expected utility is subject to a sequence of budget constraints of the form

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 P_{F,t}(j)C_{F,t}(j)dj + E_t\{O_{t,t+1}D_{t+1}\} \leqslant D_t + W_tN_t + T_t, \qquad (10)$$

where $P_{H,t}(j)$ is the price of domestic good j and $P_{F,t}(j)$ is the price of imported good j expressed in home currency. D_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t (including firm shares), W_t is the nominal wage rate, and T_t is lump-sum transfers/taxes. $O_{t,t+1}$ is the stochastic discount factor for one-period-ahead nominal payoffs relevant to the domestic household.

Given the constant elasticity of the substitution aggregator for $C_{H,t}$ and $C_{F,t}$, the optimal allocation for good j is provided by the following demand functions:

$$C_{H,t}(j) = (\frac{P_{H,t}(j)}{P_{H,t}})^{-\xi}\, C_{H,t} \text{ and } C_{F,t}(j) = (\frac{P_{F,t}(j)}{P_{F,t}})^{-\xi}\, C_{F,t}. \qquad (11)$$

Note that the above functions define the quantities consumed for each type of good, where $P_{H,t}$ and $P_{F,t}$ are the domestic and foreign price indices expressed in domestic currency. $P_{H,t}$ and $P_{F,t}$ are then given by the following expressions:

$$P_{H,t} = [\int_0^1 (P_{H,t}(j))^{1-\xi} dj]^{\frac{1}{1-\xi}}, \text{ and } P_{F,t} = [\int_0^1 (P_{F,t}(j))^{1-\xi} dj]^{\frac{1}{1-\xi}}. \qquad (12)$$

Combining (11) and (12), we obtain $\int_0^1 P_{H,t}(j)C_{H,t}(j)dj = P_{H,t}C_{H,t}$, and $\int_0^1 P_{F,t}(j)C_{F,t}(j)dj = P_{F,t}C_{F,t}$.

Similarly, it can be shown that the optimal allocations between domestic and imported goods are provided by relations

$$C_{H,t} = (1-\alpha)(\frac{P_{H,t}}{P_t})^{-\theta} C_t, \text{ and } C_{F,t} = \alpha(\frac{P_{F,t}}{P_t})^{-\theta} C_t$$

where $P_t \equiv [(1-\alpha)(P_{H,t})^{1-\theta} + \alpha(P_{F,t})^{1-\theta}]^{\frac{1}{1-\theta}}$ is the overall consumer price index (CPI), or, in loglinearized form, $\hat{p}_t = (1-\alpha)\,\hat{p}_{H,t} + \alpha\,\hat{p}_{F,t}$. Accordingly, total consumption expenditures for the domestic household are given by

$$P_{H,t}\, C_{H,t} + P_{F,t}\, C_{F,t} = P_t\, C_t. \tag{13}$$

Substituting this relationship back into (10), we can rewrite the intertemporal budget constraint as

$$P_t\, C_t + E_t\{O_{t,t+1} D_{t+1}\} \leqslant D_t + W_t N_t + T_t. \tag{14}$$

To solve the household's optimization problem, we introduce the following functional form for the utility function[4] $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}$. This yields the following set of first order conditions. First, the intratemporal optimality condition $C_t^{\sigma} N_t^{\phi} = \frac{W_t}{P_t}$ states that at any period of time t, the marginal utility of consumption is equal to the marginal value of labour. On the other hand, intertemporal optimization (for all states and dates) implies the following Euler equation with regards to consumption:

$$E_t O_{t,t+1} = \beta(\frac{C_{t+1}}{C_t})^{-\sigma}(\frac{P_t}{P_{t+1}}). \tag{15}$$

Let $R_t = 1/E_t O_{t,t+1}$ define the gross return on a riskless one-period discount bond paying off one unit of domestic currency in $t+1$. Equation (15) can easily be rewritten as a standard Euler equation

$$\beta R_t E_t\{(\frac{C_{t+1}}{C_t})^{-\sigma}(\frac{P_t}{P_{t+1}})\} = 1 \tag{16}$$

which, in loglinearized form, yields, $\hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma}(r_t - E_t \hat{\pi}_{t+1} - \rho)$ where $\rho \equiv -\log\beta$ is the time discount factor.

[4]Then the Lagrangian expression for this problem is given by $\underset{C_t,N_t,D_{t+1}}{Max} E_0\{\sum_{t=0}^{\infty}\beta^t[\frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\phi}}{1+\phi} + \lambda_t(D_t + W_t N_t + T_t - P_t C_t - E_t(O_{t,t+1} D_{t+1}))]\}$.

We assume households in the rest of the world face the same optimization problem as the one outlined above where influence from the domestic economy is negligible; i.e. relative to the ROW economy, the size of the small-open economy is negligible.[5]

Inflation, terms of trade and the real exchange rate

This section sets out the relationships between inflation, terms of trade and the exchange rate. We assume that the law of one price holds for all goods (including imported goods), at all times implying that

$$P_{F,t}(j) = \epsilon_t P^F_{F,t}(j) \text{ for all } j \in [0,1], \tag{17}$$

where ϵ_t is the bilateral nominal exchange rate[6] and $P^F_{F,t}(j)$ is the price of good (j) produced in a foreign country, as expressed in terms of the foreign currency.

Substituting (12) back into (17) yields the following expression for the foreign price index $P_{F,t} = \epsilon_t [\int_0^1 (P^F_{F,t}(j))^{1-\xi} dj]^{\frac{1}{1-\xi}}$. Similarly, if we define the foreign price index as $P^*_t = [\int_0^1 (P^F_{F,t}(j))^{1-\xi} dj]^{\frac{1}{1-\xi}}$, we can write the relation between the home price of imported goods and the foreign price index in loglinearized form around a steady-state as

$$\hat{p}_{F,t} = \hat{e}_t + \hat{p}^*_t. \tag{18}$$

In addition, using the terms-of-trade definition $S_t = \epsilon_t P^*_t / P_{H,t}$, and loglinearizing around a symmetric steady-state yields

$$\hat{s}_t = \hat{e}_t + \hat{p}^*_t - \hat{p}_{H,t}. \tag{19}$$

Thus terms of trade may be defined as the price of foreign goods per unit of home good. Since, by definition, the real exchange rate (in loglinearized form) is given by $\hat{q}_t = \hat{e}_t + \hat{p}^*_t - \hat{p}_t$, substituting

[5] This assumption allows us to treat the ROW economy as a closed economy.

[6] The price of foreign-country currency in terms of domestic currency. An increase in ϵ_t coincides with an appreciation of domestic currency.

into equation (19) yields the following relation between the real exchange rate and the terms of trade given the price levels:

$$\hat{q}_t = \hat{s}_t + \hat{p}_{H,t} - \hat{p}_t. \tag{20}$$

On the other hand, using the definition of the price indices, it can be shown that

$$\frac{P}{P_H}\hat{p}_t - \frac{P}{P_H}\hat{p}_{H,t} = [(1-\alpha) + \alpha(S)^{1-\theta}]^{\frac{1}{1-\theta}} S^{1-\theta}\alpha\hat{s}_t. \tag{21}$$

Furthermore, assuming that purchasing power parity (PPP) holds in the steady-state, i.e. $S = \frac{P_F}{P_H} = 1$, and combining this equation with (19) and (21), we can write the relation between the domestic price level and the CPI as follows:

$$\hat{p}_t - \hat{p}_{H,t} = \alpha\hat{s}_t. \tag{22}$$

As a result, we arrive at identity-linking CPI inflation ($\hat{\pi}_t$), domestic inflation ($\hat{\pi}_{H,t}$) and the change in the term of trade :

$$\hat{\pi}_t = \hat{\pi}_{H,t} + \alpha\Delta\hat{s}_t. \tag{23}$$

The difference between the CPI and domestic inflation is proportional to the change in the terms of trade, and the coefficient of proportionality increases with the degree of openness, α. Furthermore, substituting (22) back into (20) yields an expression for the real exchange rate as a function of terms of trade, i.e.

$$\hat{q}_t = (1-\alpha)\hat{s}_t, \tag{24}$$

which establishes a relation between real exchange rate and terms of trade, depending on the degree of openness of the SOE.

International Risk Sharing

In our model, we assume that a complete securities market actually exists in the world, where the expected nominal return from risk-free bonds, in domestic currency terms, must be the same as the expected domestic-currency return from foreign bonds, i.e. $E_t O_{t,t+1} = E_t O^*_{t,t+1} \frac{\epsilon_t}{\epsilon_{t+1}}$. The Euler equation also holds for the foreign representative household and must equate the intertemporal optimality condition for the domestic household.[7] This yields a relationship between the domestic and foreign level of consumption in terms of (log) real exchange rate (Q_t), i.e.

$$C_t = Q_{t+1}^{-\frac{1}{\sigma}} \frac{C_{t+1}}{C^*_{t+1}} C^*_t Q_t^{\frac{1}{\sigma}}. \tag{25}$$

Finally, replacing C_{t+1} and C^*_{t+1} with their respective expression yields the following optimal allocation for the imported good:

$$C^*_t = \alpha Q_t^{-\theta} C_t. \tag{26}$$

Therefore, the relation in (25) can be rewritten as $C_t = \Phi_o C^*_t Q_t^{\frac{1}{\sigma}}$, where Φ_o depends on the initial condition of the country's asset position. If we assume symmetric initial conditions between home and foreign country, with zero foreign asset holdings for the small-open economy, without loss of generality we obtain $\Phi_o = 1$ so that the loglinearized form leads to

$$\hat{c}_t = \hat{c}^*_t + \frac{1}{\sigma} \hat{q}_t. \tag{27}$$

Substituting (24) back into (27) then yields

$$\hat{c}_t = \hat{c}^*_t + \frac{1-\alpha}{\sigma} \hat{s}_t. \tag{28}$$

which links both consumption variables to the terms of trade.

Uncovered Interest Parity

The assumption of complete securities markets points to another important relationship, the uncovered interest parity (UIP) condition. Using the previous Euler equation, which also holds for foreign

[7] Using the fact that $E_t O_{t,t+1} = \beta E_t \{ (\frac{C^*_{t+1}}{C^*_t})^{-\sigma} (\frac{P^*_t}{P^*_{t+1}}) (\frac{\epsilon_t}{\epsilon_{t+1}}) \}$, combining this equation with its domestic counterpart, substituting in $Q_t = \epsilon_t P^*_t / P_t$ and rearranging terms to get the next equation.

households, i.e., $\beta R^*_t E_t\{(\frac{C^*_{t+1}}{C^*_t})^{-\sigma}(\frac{P^*_t}{P^*_{t+1}})\} = 1$, or put another way,

$$R^{*-1}_t = \beta E_t\{(\frac{C^*_{t+1}}{C^*_t})^{-\sigma}(\frac{P^*_t}{P^*_{t+1}})\} \tag{29}$$

and substituting (29) back into the Euler equation yields the price of a riskless bond dominated in foreign currency as

$$R^{*-1}_t \epsilon_t = E_t\{O_{t,t+1}\epsilon_{t+1}\}. \tag{30}$$

Since, by definition, $R^{-1}_t = E_t O_{t,t+1}$, (30) implies that $E_t\{O_{t,t+1}[R_t - R^*_t(\epsilon_{t+1}/\epsilon_t)]\} = 0$, loglinearizing around the perfect-foresight steady-state yields the asset pricing equation for nominal bonds, which implies that the interest rate differential is related to the expected exchange rate depreciation

$$\hat{r}_t - \hat{r}^*_t = E_t\{\Delta\hat{e}_{t+1}\}, \tag{31}$$

where $\hat{e}_t$ is the deviation of the nominal exchange rate from its steady-state value. Thus, the UIP condition for the nominal exchange rate holds in equilibrium, meaning that the risk premium is assumed to be constant in the steady-state.

Monetary Policy

To close the model, we assume that the central bank sets the nominal interest rate following a Taylor-type interest-rate rule. In its simple version introduced by the influential work by Taylor (1993), an interest rate feedback from output and inflation is used to approximate monetary policy. Recently Woodford (2000) demonstrated that the interest rate rule is consistent with nominal demand determinacy for forward-looking models. In addition, in an open-economy model, the exchange rate is affected by the difference between domestic and foreign nominal interest rates and expected future exchange rates, via an interest rate parity condition (Svensson, 1998). The real exchange rate will then affect the relative price of domestic and foreign goods, which in turn affects both domestic and foreign demand for domestic goods and hence contributes to movements in CPI inflation. Likewise, the exchange rate affects the domestic currency prices of imported final goods included in the CPI price. In this way, monetary policy can affect both the CPI price and the CPI inflation rate. Consequently, when analyzing our model under HT targeting, we consider a monetary rule that incorporates both the price level and the inflation rate.

In the present paper, we analyze the macroeconomic implications of three alternative monetary-policy regimes for the small-open economy: a policy that aims at fully stabilizing CPI inflation (IT), a policy that stabilizes CPI price level (PT) and a policy that combines price-level and inflation targeting (HT).

As in Galí and Monacelli (2005), we assume that the world monetary authority succeeds in fully stabilizing world prices and the output gap; hence, we assume $\hat{y}_t^* = \pi_t^* = 0$ for all t which is an optimal policy for the closed economy under our assumptions.[8]

Equilibrium Determination

Aggregate Demand

World Output and Consumption

Combining the market clearing condition for the ROW economy, $\hat{y}_t^* = \hat{c}_t^*$, with the Euler equation for the foreign household's consumption, $\hat{c}_t^* = E_t\hat{c}_{t+1}^* - \frac{1}{\sigma}(r_t^* - E_t\hat{\pi}_{t+1}^* - \rho)$, leads to a version of the new IS equation in the case of sticky-price models:

$$\hat{y}_t^* = E_t\hat{y}_{t+1}^* - \frac{1}{\sigma}(r_t^* - E_t\hat{\pi}_{t+1}^* - \rho). \tag{32}$$

This IS equation shows that the foreign output is related negatively to the world interest rate and positively to expected foreign CPI inflation.

[8] The reader is referred to Clarida et al. (2000 and 2002) and Galí and Monacelli (2005) for the derivation of such a rule and its optimality for closed economy version of the model.

Small-Open-Economy Output, Consumption and Trade Balance

Market clearing for domestic goods requires that $Y_t(i) = C_{H,t}(i) + C^*_{H,t}(i)$, where $Y_t(i)$, $C_{H,t}(i)$ and $C^*_{H,t}(i)$ are, respectively, the production, home and foreign demand for home produced good i. Moreover, based on preference symmetry between the home and the foreign country, it can be shown that

$$C^*_{H,t} = \alpha(\frac{P_{H,t}(i)}{P_{H,t}})^{-\xi}(\frac{P_{H,t}}{\epsilon_t P^F_{F,t}})^{-\xi}(\frac{P^F_{F,t}}{P^*_t})^{-\theta}C^*_t. \tag{33}$$

Substituting (33) back into in the market clearing condition above, we get

$$Y_t(i) = (\frac{P_{H,t}(i)}{P_{H,t}})^{-\xi}\{(1-\alpha)(\frac{P_{H,t}}{P_t})^{-\theta}C_t + \alpha(\frac{P_{H,t}}{\epsilon_t P^F_{F,t}})^{-\xi}(\frac{P^F_{F,t}}{P^*_t})^{-\theta}C^*_t\}$$

for all $i \in [0,1]$ and for all t.

Using the fact that $S_t \equiv \epsilon_t P^F_{F,t}/P_{H,t}$, the aggregate output can be shown to reduce to

$$Y_t = (\frac{P_{H,t}}{P_t})^{-\theta}C_t[(1-\alpha) + \alpha S_t^{\xi-\theta}Q_t^{\theta-\frac{1}{\sigma}}]. \tag{34}$$

Log-linearizing (34) while making use of $\hat{p}_t - \hat{p}_{H,t} = \alpha\hat{s}_t$ yields

$$\hat{y}_t = \hat{c}_t + \alpha\xi\hat{s}_t + \alpha(\theta - \frac{1}{\sigma})\hat{q}_t. \tag{35}$$

Equation (35) states that the relation between output and consumption in terms of the exchange rate and terms-of-trade variables is governed by the degree of openness of the economy.

Furthermore, notice that by using $\hat{q}_t = (1-\alpha)\hat{s}_t$, expression (35) can be rewritten as

$$\hat{y}_t = \hat{c}_t + \frac{\alpha\omega}{\sigma}\hat{s}_t, \tag{36}$$

where $\omega = \xi\sigma + (\sigma\theta - 1)(1-\alpha)$. Using the fact that $\hat{c}_t = \hat{y}^*_t + (\frac{1-\alpha}{\sigma})\hat{s}_t$, equation (36) becomes

$$\hat{y}_t = y^*_t + \frac{1}{\sigma_\alpha}\hat{s}_t \tag{37}$$

where $\sigma_\alpha = \sigma/[(1-\alpha)+\alpha\omega]$, and the subscript in σ_α is meant to emphasize the dependence of this parameter on the degree of openness of the economy (α). Finally we can compute a version of the new IS equation for the SOE by combining Euler equation (16) with (23) and (36), which yields $\hat{y}_t = E_t\{\hat{y}_{t+1}\} - \frac{1}{\sigma}(\hat{r}_t - E_t\{\hat{\pi}_{H,t+1}\} - \rho) - \frac{\alpha\omega}{\sigma}E_t\{\hat{s}_{t+1}\}$.

This leads to a difference equation for output related to the domestic interest rate, world output and domestic inflation:

$$\hat{y}_t = E_t\{\hat{y}_{t+1}\} - \frac{1}{\sigma_\alpha}(\hat{r}_t - E_t\{\hat{\pi}_{H,t+1}\} - \rho) + \alpha(\omega - 1)E_t\{\Delta\hat{y}^*_{t+1}\}. \tag{38}$$

This SOE equation is different from its closed-economy version because it depends on the small economy's degree of openness and on foreign output.[9]

Moreover, net exports (nx) are related to domestic output in terms of steady-state output (Y) through the following equation

$$nx_t = (\frac{1}{Y})(Y_t - \frac{P_t}{P_{H,t}}C_t). \tag{39}$$

Combining the linearized version of (39) with (22), (28) and (36) yields

$$\hat{nx}_t = (1 - \Lambda)(\hat{y}_t - \hat{y}^*_t)$$

where $\Lambda = \sigma_\alpha[(1 - \alpha) + \alpha\sigma]/\sigma$. The relationship between net exports and the output differential is ambiguous and depends on the value of Λ. If $-1 < \Lambda < 1$, a positive output differential generates a trade surplus favourable to the small-open economy and with $\Lambda > 1$ or $\Lambda < -1$ the trade surplus is favourable to the foreign country. Following Galí and Monacelli (2005), we need $-1 \leq \Lambda \leq 1$ to satisfy the Marshall-Lerner conditions.[10]

Appendix B: Deriving the New Keynesian Phillips Curve (NKPC)

Price stickiness is the only source of suboptimality in the equilibrium allocation. Indeed, as shown by Galí and Monacelli (2005), the employment subsidy neutralizes the market power distortion and

[9] It's easy to see that with $\alpha = 0$, we can obtain the closed-economy version.

[10] The Marshall-Lerner conditions apply if and only if the sum of the import and export elasticities is greater than one.

by not assigning any explicit value to monetary holding balances, the monetary distortion that would pull monetary policy towards the Friedman rule is eliminated. Inflation inertia is also introduced in the model by the price behaviour. The resulting model is then consistent with what has been termed the NKPC. The determination of the real marginal cost as a function of domestic output and foreign output is complex due to the wedge between some aggregate variables, namely output versus consumption and domestic price versus consumer price indexes. We indeed have

$$\begin{aligned} \widehat{mc}_t &= -\nu + \hat{w}_t - \hat{p}_{H,t} - \hat{a}_t \qquad (40) \\ &= -\nu + \phi\hat{y}_t + \sigma\hat{y}_t^* + \hat{s}_t - (1+\phi)\hat{a}_t, \end{aligned}$$

where the last equality makes use of (3) and (28). According to (40), real marginal cost is increasing as concerns the terms of trade, domestic output and world output and is decreasing with regards to technology. Hence, the wealth and employment effects on real wages, combined with the changes in the product wage and then the impacts on real wages lead to changes in marginal cost through its direct effect on labour productivity. It follows from equation (37) that in this case real marginal cost is given by

$$\widehat{mc}_t = -\nu + (\sigma_\alpha + \phi)\hat{y}_t + (\sigma - \sigma_\alpha)\hat{y}_t^* - (1+\phi)\hat{a}_t. \qquad (41)$$

In what follows, we focus on equation (7) to derive a NKPC representation for small-open economy in terms of the output gap and domestic inflation given our price schemes. Let's define the output gap[11] $\bar{x}_t$ as the deviation of domestic output $\hat{y}_t$ from its 'natural' level $\bar{y}_t$. Formally, $\bar{x}_t = \hat{y}_t - \bar{y}_t$, where natural output is computed by imposing the restriction $\widehat{mc}_t = -\mu$ for all t in equation (41) and solving for domestic output, i.e. $-\mu = -\nu + (\sigma_\alpha + \phi)\bar{y}_t + (\sigma - \sigma_\alpha)\hat{y}_t^* - (1+\phi)\hat{a}_t$,

[11] In our model, we have to handle three definitions of output: a measure of output, natural output (which we get in an economy with no imperfection or nominal rigidity) and finally the output gap, which is the difference between the output and the natural output.

which, after some algebraic manipulations, yields the natural output level as

$$\bar{y}_t = \Omega + \Gamma \hat{y}_t^* + \Psi \hat{a}_t, \tag{42}$$

where Ω =$(\nu - \mu)/(\sigma_\alpha + \phi)$, $\Gamma = (\sigma_\alpha - \sigma)/(\sigma_\alpha + \phi)$ and $\Psi = (1 + \phi)/(\sigma_\alpha + \phi)$. Equation (42) states that the natural output for the small-open economy is determined by world output and productivity, as well as domestic markup. In addition, we can derive a relationship between real marginal cost and the output gap according to

$$\stackrel{\wedge}{mc}_t = -\nu + (\sigma_\alpha + \phi)\bar{x}_t + (\sigma_\alpha + \phi)(\Omega + \Gamma y_t^* + \Psi \hat{a}_t) + (\sigma - \sigma_\alpha)\hat{y}_t^* - (1 + \phi)\hat{a}_t$$

where natural output has been substituted for its value in (42). By rearranging terms, we get

$$\stackrel{\wedge}{mc}_t = (\sigma_\alpha + \phi)\bar{x}_t, \tag{43}$$

which we can combine with equation (7) to derive a NKPC in terms of the output gap

$$\hat{\pi}_{H,t} = \frac{\beta}{1 + \beta\psi\gamma_p} E_t\{\hat{\pi}_{H,t+1}\} + \frac{\gamma_p}{1 + \beta\psi\gamma_p} \hat{\pi}_{H,t-1} + \delta \bar{x}_t, \tag{44}$$

where $\delta = \kappa(\sigma_\alpha + \phi)$. Notice that with the degree of openness (α) and the coefficient of price indexation γ_p set to zero (i.e. $\alpha = 0$ and $\gamma_p = 0$), equation (44) reverts to the standard, purely forward-looking, NKPC. The relation (44) also makes it clear that the standard formulation of NKPC based on the output gap assumes no price indexation to past inflation, and hence there is no inflation inertia in the model.

The equilibrium dynamics for the small-open economy in terms of output gap and domestic inflation can be completed by writing a version of the IS equation in terms of the output gap. Indeed, by combining (38) and (42), it can be shown that

$$\bar{x}_t = E_t\{\bar{x}_{t+1}\} - \frac{1}{\sigma_\alpha}(\hat{r}_t - E_t\{\hat{\pi}_{H,t+1}\} - \rho) + \Gamma(\rho_a - 1)\hat{a}_t + \alpha(\Psi + \Theta)E_t\{\hat{y}_{t+1}^*\}, \tag{45}$$

where $\Theta = (\omega - 1)$. If we define the natural interest rate as $\bar{rr}_t \equiv \rho - \sigma_\alpha \Gamma(\rho_a - 1)\hat{a}_t + \alpha\sigma_\alpha(\Psi + \Theta)E_t\{\hat{y}^*_{t+1}\}$, where the degree of openness and the expected world output affect the natural rate of interest, then the new IS equation has the following form:

$$\bar{x}_t = E_t\{\bar{x}_{t+1}\} - \frac{1}{\sigma_\alpha}(\hat{r}_t - E_t\{\hat{\pi}_{H,t+1}\} - \bar{rr}_t). \quad (46)$$

Equation (46) relates the output gap in the forward-looking equation to the interest rate, domestic inflation and the natural interest rate.

To solve this model, we make a loglinear approximation of the equilibrium conditions around a balanced-trade, zero-inflation steady-state.[12] The dynamic properties of the model crucially depend on the monetary policy used. Indeed, with the Taylor rule, where the χ parameter takes the value 1, the persistent inflation response to a technology shock implies that this shock will have a permanent effect on price level, which will then have a unit root, mirrored by a unit root in the nominal exchange rate. In this case, and following Galí and Monacelli (2005), when targeting inflation rate, the monetary authority seeks to stabilize CPI inflation. Such a policy only requires that we set $\hat{r}_t - E_t\{\hat{\pi}_{t+1}\} = \phi_p(E_t\hat{\pi}_t) + \phi_y\bar{x}_t$, for all t. Moreover, following Woodford (1999) and Bullard and Mitra (2002), our analysis focuses on the case where ϕ_p and ϕ_y have non-negative values. Thus, the necessary and sufficient condition for a stable allocation path[13] is given by

$$\delta(\phi_p - 1) + (1 - \beta)\phi_y \neq 0. \quad (47)$$

Furthermore, we assume that the foreign country pursues an optimal policy, implying a constant foreign-price level at equilibrium.[14] The model's dynamics can be stable in this case, even with non-stationary prices. Otherwise, with $0 \leqslant \chi < 1$ the price level is I(0) and the stable allocation-path condition (47) holds at equilibrium.

[12] The markup is also assumed to be constant at steady state ($\mu = \frac{\xi}{\xi - 1}$) in order to derive the equilibrium conditions.

[13] As shown by Bullard and Mitra (2003), this condition rules out eigenvalues on the unit circle.

[14] See Galí and Monacelli (2004) for a discussion of optimal policy in the foreign-country and SOE cases.

In the following sections, we will first set the model parameters as calibrated to the Canadian economy, and before analyzing the welfare implications of each regime, compute the impulse response functions and second-moment statistics .

Derivation of NKPC in Small Open Economy with Inflation Inertia

Price setting behavior follows a modified Calvo (1983) scheme in that it allows for some inflation inertia due to partial price indexation. The dynamic of domestic price index is given by

$$P_{H,t} = [\psi(P^{b}_{H,t-1})^{1-\xi} + (1-\psi)(P^{n}_{H,t})^{1-\xi})]^{\frac{1}{1-\xi}}$$

which can be loglinearized to obtain, $\hat{p}_{H,t} = \psi\hat{p}^{b}_{H,t} + (1-\psi)\hat{p}^{n}_{H,t}$ where $\hat{p}^{b}_{H,t}$ is the time t price for backward-looking firms and $\hat{p}^{n}_{H,t}$ is the time t price for forward-looking firms.

Replacing the backward-looking price by its value we get

$$\begin{aligned}
\hat{p}_{H,t} &= \psi\gamma_p\hat{\pi}_{H,t-1} + (1-\psi)(\hat{p}^{n}_{H,t} - \hat{p}_{H,t-1}) + \hat{p}_{H,t-1} \text{ or in term of domestic inflation,} \\
\hat{\pi}_{H,t} &= \psi\gamma_p\hat{\pi}_{H,t-1} + (1-\psi)(\hat{p}^{n}_{H,t} - \hat{p}_{H,t-1}) \qquad (48)
\end{aligned}$$

Arranging terms in the last equation we can write the inflation equation as $\hat{\pi}_{H,t} = \psi\gamma_p\hat{\pi}_{H,t-1} + (1-\psi)(\hat{p}^{n}_{H,t} - \hat{p}_{H,t-1})$.

Furthermore, when setting a new price ($\hat{p}^{n}_{H,t}$), an optimizing firm will seek to maximize the current value of its dividend stream subject to the sequence of demand constraints. In aggregate form the following function is maximized

$$\underset{\hat{p}^{n}_{H,t}}{Max}\sum_{k=0}^{\infty}\psi^{k}E_t\{O_{t,t+1}Y_{t+k}(\hat{p}^{n}_{H,t} - MC^{n}_{t+k})\}$$

$$\text{subject to } Y_{t+k} \leqslant (\frac{\hat{p}^{n}_{H,t}}{\hat{p}^{n}_{H,t+k}})^{-\xi}(C_{H,t+k} + C^{*}_{H,t+k})$$

where MC^n_{t+k} is the nominal marginal cost, ξ is the elasticity of substitution among goods and $C_{H,t+k}$ and $C^*_{H,t+k}$ are, respectively, domestic and foreign consumption of domestic goods. The first order condition can, then, be computed as

$$\sum_{k=0}^{\infty} \psi^k E_t\{O_{t,t+1} Y_{t+k}(\hat{p}^n_{H,t} - \frac{\xi}{1-\xi} MC^n_{t+k})\} = 0.$$

The decision rule for $\hat{p}^n_{H,t}$ follows after some algebraic manipulation and loglinearization around steady-state

$$\hat{p}^n_{H,t} - \mu = (1-\beta\psi) \sum_{k=0}^{\infty} (\beta\psi)^k E_t\{\hat{mc}_{t+k} + \hat{p}_{H,t+k}\} \tag{49}$$

where $\mu = \frac{\xi}{1-\xi}$ is the steady-state markup. Splitting up the summation in LHS into two terms. One at date t (with $k = 0$) and the other at $t+1$ (with $k = 1 \longmapsto \infty$). This leads to write (49) as

$$\hat{p}^n_{H,t} = \mu + (1-\beta\psi)[\hat{mc}_t + \hat{p}_{H,t}] + (1-\beta\psi)(\beta\psi) \sum_{k=0}^{\infty} (\beta\psi)^k E_t\{\hat{mc}_{t+k+1} + \hat{p}_{H,t+k+1}\}$$

which can be rearranged by writing (49) at date $t+1$, replacing the term in LHS by its value (i.e. $\hat{p}^n_{H,t+1} - \mu$) and subtracting $\hat{p}_{H,t-1}$ from both side (and dropping constant term) of this equation to get

$$\hat{p}^n_{H,t} - \hat{p}_{H,t-1} = \hat{\pi}_{H,t} + \beta\psi(\hat{p}^n_{H,t+1} - \hat{p}_{H,t}) + (1-\beta\psi)\hat{mc}_t. \tag{50}$$

Using (48) we can compute the time $t+1$ value of this equation as $\hat{p}^n_{H,t+1} - \hat{p}_{H,t} = \frac{1}{1-\psi}\hat{\pi}_{H,t+1} - \frac{\psi\gamma_p}{1-\psi}\hat{\pi}_{H,t}$. Combining the two previous results (48 and 50), it follows that

$$\hat{\pi}_{H,t} = \psi\gamma_p\hat{\pi}_{H,t-1} + (1-\psi)[\hat{\pi}_{H,t} + \beta\psi(\frac{1}{1-\psi}\hat{\pi}_{H,t+1} - \frac{\psi\gamma_p}{1-\psi}\hat{\pi}_{H,t})] + (1-\beta\psi)\hat{mc}_t$$

from which it can follows that

$$\hat{\pi}_{H,t} = \frac{\beta}{1+\beta\psi\gamma_p} E_t\{\hat{\pi}_{H,t+1}\} + \frac{\gamma_p}{1+\beta\psi\gamma_p}\hat{\pi}_{H,t-1} + \kappa\hat{mc}_t.$$

Moreover, the real marginal cost in term of output gap is given by $\hat{mc}_t = (\sigma_\alpha + \phi)\bar{x}_t$. Collecting those results we can write a NKPC in small open economy model with partial indexation and sticky price behavior a la Calvo as follows:

$$\hat{\pi}_{H,t} = \frac{\beta}{1+\beta\psi\gamma_p}E_t\{\hat{\pi}_{H,t+1}\} + \frac{\gamma_p}{1+\beta\psi\gamma_p}\hat{\pi}_{H,t-1} + \delta\bar{x}_t$$

which is equation (45) in the paper.

Appendix B

Small Open Economy DSGE Model Equations

- New IS

$$\bar{x}_t = E_t\bar{x}_{t+1} - \frac{1}{\sigma_\alpha}\hat{r}_t + \frac{1}{\sigma_\alpha}\overline{rr}_t + \frac{1}{\sigma_\alpha}E_t\hat{\Pi}_{H,t+1} \tag{51}$$

- Phillips Curve

$$\hat{\pi}_{H,t} = \frac{\beta}{1+\beta\psi\gamma_p}E_t\{\hat{\pi}_{H,t+1}\} + \frac{\gamma_p}{1+\beta\psi\gamma_p}\hat{\pi}_{H,t-1} + \delta\bar{x}_t \tag{52}$$

- Natural Interest Rate

$$\overline{rr}_t = -\sigma_\alpha\Gamma(\rho_a - 1)\hat{a}_t + \alpha\sigma_\alpha(\Psi + \Theta)E_t\{\hat{y}^*_{t+1}\} \tag{53}$$

- Monetary Policy (HT)

$$\hat{r}_t = E_t\hat{\Pi}_{t+1} + \phi_p(\hat{p}_t - \chi\hat{p}_{t-1}) + \phi_y\bar{x}_t + \epsilon_{r,t} \tag{54}$$

- Output Equation

$$\hat{y}_t = [(1-\alpha)(\sigma\theta - 1)]\hat{c}_t - \alpha(\sigma\theta - 1)\hat{y}^*_t + \alpha\xi\hat{s}_t \tag{55}$$

- UIP Condition

$$\hat{r}_t = \hat{r}^*_t + E_t\hat{e}_{t+1} - \hat{e}_t \tag{56}$$

- CPI and Domestic Inflation

$$\hat{\Pi}_t = \hat{\Pi}_{H,t} + \alpha\hat{s}_t - \alpha\hat{s}_{t-1} \tag{57}$$

- Term of Trade and Nominal Exchange Rate

$$\hat{s}_t = \hat{s}_{t-1} + \hat{e}_t - \hat{e}_{t-1} + \hat{\Pi}_t^* - \hat{\Pi}_{H,t} \tag{58}$$

- Domestic Price Level (P_H)

$$\hat{p}_{H,t} = \hat{p}_{H,t-1} + \hat{\Pi}_{H,t} \tag{59}$$

- PCI Price Level (P)

$$\hat{\Pi}_t = \hat{p}_t - \hat{p}_{t-1} \tag{60}$$

- Euler Equation

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}\hat{r}_t + \frac{1}{\sigma}E_t\hat{\Pi}_{t+1} \tag{61}$$

- Terms of Trade

$$\widehat{nx}_t = (1-\Lambda)y_t - (1-\Lambda)y_t^* \tag{62}$$

- Risk Sharing

$$\hat{c}_t = y_t^* + \frac{1-\alpha}{\sigma}\hat{s}_t \tag{63}$$

- Marginal Costs Equation (ROW)

$$\widehat{mc}_t^* = (\phi+\sigma)y_t^* + (1+\phi)\hat{a}_t^* \tag{64}$$

- Phillips Curve (ROW)

$$\hat{\Pi}_t^* = \beta E_t\hat{\Pi}_{t+1}^* + \kappa\widehat{mc}_t^* \tag{65}$$

- Optimal Monetary Policy (ROW)

$$r_t^* = \rho_{r^*}E_t\hat{\Pi}_{t+1}^* + \phi_{a^*}\hat{a}_t^* \tag{66}$$

- Home Technology Shock

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \epsilon_{a,t} \tag{67}$$

- Foreign Technology Shock

$$\hat{a}_t^* = \rho_{a^*}\hat{a}_{t-1}^* + \epsilon_{a,t}^* \tag{68}$$

Monetary Policy (Other than HT):

- Price Targeting (PT)

$$\hat{p}_t = 0 \tag{69}$$

- Inflation Targeting (IT)

$$\hat{r}_t = \rho + \phi_\pi \hat{\Pi}_t + \phi_y \bar{x}_t \ + \epsilon_{r,t} \tag{70}$$

Shocks in this small open economy: $\epsilon_{a,t}, \epsilon_{a,t}^*$ and $\epsilon_{r,t}$.

Table 1: Model Parametrization

Parameters	μ	ϕ	ξ	θ	σ	α	ψ	χ	ϕ_p	ϕ_π	ϕ_y	ρ_A	σ_A	ρ_{A*}	σ_{y*}
Values assigned	1.2	3	6	1.5	1.5	0.4	0.75	0.55	0.5	0.5	0.5	0.66	1%	0.76	1%